

Same Symbols, Different Representation: The Semiotics of Linguistics and Mathematical Notations

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Abstract

This paper focuses on the disciplinarity of mathematics and linguistics. Its objective is to explore the same symbols in use by the two disciplines. Within several disciplines and professional fields are found the applications of similar techniques and methods in use. Such operations are applied to bring the abstractions of such disciplinarity close to concreteness. It has been discovered that some fields make use of the same symbols but have different representations in the varying fields. The work through semiotic theory investigates the different representations of similar symbols found in linguistics and mathematics (maths). Our finding is that there are many similar symbols in use by both linguistics and mathematics. They include: the cross/positive (+) symbol, negative/minus sign (-), the 'x' symbol, the symbols of inequality- > < (greater & less than), the superscript symbol (^{1, mmm}) the equivalent/equal sign (=), structures, shapes etc.. The cross/plus sign in mathematics represents addition whose summative product is written together while in linguistics (syntactic transformation), it is an insertion of word/words in a phrase/sentence whose output cannot be written together. In phonology, a positive/cross sign shows the presence of class features. In mathematics, the superscripts are used to show the power to which a number can be raised. In linguistics, they represent the levels of phrasal categories (the maximal, the intermediate and the lexical). The 'x' (multiplication) sign is used for scaling operation in mathematics while in linguistics, it stands for any lexical category symbol (N, V, A, P etc.). It is also used in projection operation. The work therefore, finds out that most of the symbols though similar have different representations in the two disciplines.

1. Introduction

In the study of interdisciplinarity, attempts are made to lend some techniques, rules, signs in one discipline to the other. In other words, both have interest in application of some similar concepts that even the simplest elements in one can be put to use in the other. Many fields use similar signs, symbols or even techniques in their operations but in many cases their representations are not the same. For instance, the symbol 'minus' is used in mathematics as a subtraction operator, a negative number when placed in front or even a unary operator that acts as an instruction but in chemistry, the minus sign (not en dash) is

used for a single covalent bond between two atoms. We shall limit our study to the fields of mathematics and linguistics. Briefly, we shall discuss the two disciplines to help us capture some similar methods, rules, symbols and signs etc. used by them to ascertain whether they have the same representation in the disciplines studied.

Language is the major medium through which human beings communicate and interact. Robbins (1980) observes that language is the use of finite arbitrary vocal symbols combined according to rules of the grammar for the purpose of communication. The scientific study of language is termed linguistics. Lyons (1968) postulates that linguistics is an academic discipline that studies language scientifically as a means of communication used primarily by human beings. Linguistics is, therefore, a science that engages itself with the application of those criteria which other scientists in other scientific disciplines apply in the analysis of their data. The linguist aims at consistency, explicitness and precision. According to Anagbogu, Mbah and Eme (2010), the linguist handles his data the same way a biologist or physical scientist handles his specimen (data). His operations are systematic, methodological, objective and organised. The major scope of linguistics includes phonology, morphology, syntax and semantics

The term mathematics according to Pepin (2007) is a science dealing with the logic of quantity, shape, and arrangement. He refers to mathematics as the knowledge centered on such concepts as structural, quantity, space, shapes, addition, and deduction together with the academic discipline that studies them. Delvin (1997) claims that mathematics is the science of patterns whether found in numbers, space, computers, imaginary abstractions or elsewhere. The use of these concepts actualizes the full set of mathematical theories into practical and intellectual activities, induction, deduction, arguing, calculating, defining, giving premises etc. Gold (2007) argues that when it is defined this way, it involves a pragmatic understanding of language and communication. This implies that in the process of using the above concepts for problem solving, there are hardly any that does not play a role in the other disciplines. Anagbogu, Mbah and Eme (2008) observe that if one should look at the blackboard in a serious linguistics' class one will find all sorts of symbols, both those used in logic, philosophy and in mathematics. Kait (2005) approaching the definition of mathematics from a philosophical and pedagogical aspects, postulates that maths is an essentially linguistic activity characterised by the association of words with precise meaning. These views imply that some similar operators and techniques are used by both disciplines (linguistics and mathematics). In this regard, Niss (2003) observes that the scope and complexity of modern mathematics requires a very fine analysis of the formal language in which meaningful mathematical statements may be formulated and perhaps be proved true or false. The view therefore, argues that many contradictions, false calculation etc. in mathematics results from inconsistent and imprecise use of language. On the other hand, Delvin (1994) indicates that in order to capture some of the abstract patterns and structure of mind and language, linguists sometimes find it necessary to make use of mathematics or at least mathematical notations and techniques. This paper through semiotic theory examines the symbols in use in linguistics and mathematical notations. It investigates the different representations of the same symbols used by both. The few cases of same representation will also be noted.

2.0. Semiotics

Semiotics originates from the Greek word 'sema' which refers to theory/science and analysis of signs, sign systems and their meanings especially those involved in communication between human beings in different social cultures. Chandler (2007) notes that the term 'σημειωτικός, (*sēmeiōtikos*)' in Greek was used in the 16th century by Robert Scott, a Greek scholar to mean "observant of signs" and it was first used in English by Henry Stubbes in a very precise sense to refer to the branch of medical science that concerns itself with the interpretation of signs. Asher (1994) postulates that semiotics in the broadest sense is the study of basic human activity of creating meaning, signs- verbal or non-verbal, natural or artificial etc. which carry meaning. It is the study of sign structures and sign processes. Katie (1989:416) contends that the field of semiotics covers a large area of learning such as: verbal language in both speech and writing, non-verbal communication system such as gesture and body movement, codes of proximity, dress, mass media etc. According to him, it overlaps with other disciplines as communication theory, linguistics, sociolinguistics and semantics. Bathes (1967) upholds that almost anything in the society as a significant sign meaningful to the speech community. This implies that an individual can be assessed and assigned a social group or class by the dress or even the hair style worn. Ken-Maduako (2010:2) observes that "semiotics is a broad area of linguistic enquiry which involves the study of all possible signaling systems in the world and their function in human perception of meaning". Crystal (1998:399) says, "it is concerned with different modes of communication, sight, sound, smell, touch and taste in different situational contexts of dance, film politics, food and clothing". Semiotics is categorised into sub-fields such as semiotics of the structure, social semiotics, visual semiotics etc.

Social semiotics is a branch of the field of semiotics which studies human signifying practices in specific social and cultural circumstances, and which tries to explain meaning-making as a social practice. The term was first used by a linguistic theorist, Micheal Halliday in his book *–Language as social semiotics* published (1978). His ideology was prompted by his interest to show a strong relationship between language and the society. He therefore, extended the narrow focus on written language in linguistics. Thibault (1991) contends that social semiotics is the investigation into the social dimensions of meaning, including the potentiality of human processes of signification and interpretation in shaping individuals and societies. Social semiotics focuses on social meaning-making practices of all types, whether visual, verbal or written, gestural and musical resources of communication (Kress and Leeuwen 2001). Implicitly, this branch of semiotics sees linguistic and non-linguistic signs as tools employed by people to create meaning.

Linguistic semiotics is often divided into three branches: semantics which studies the relation between signs and the things to which they refer; what they denote or their meaning; Syntax investigates relations among signs in formal structures while Pragmatics accounts for relation between signs and the effects they have on the people who use them. Semiotics was taken further into structure by Ferdinand de Saussure who defined semiotics as "the science of the life of signs in society". Structuralist semiotics in the

Saussure's tradition focused primarily on theories of semiotic systems and structures which are constantly static. Ferdinand de Saussure's theories include the *langue* and *parole*. This informs his definition of linguistics as a specific science whose object is the language system; the closely interrelated structure of elements that are different from the individual use of language (Saussure 1983). In support of this view, Claude (1986) postulates that language is the semiotic system per excellence, it cannot but signify and exists only through signification. Social semiotics tries to account for the variability of semiotic practices of the *parole*. From the Saussurean discovery of the fact that the "codes" of language and communication are formed by social processes, we can contend that social semiotics has relationship with pragmatics and sociolinguistics, cultural studies and discourse analysis. Pierce (1958), a great semiotian of the nineteenth century identifies three types of signs: the symbol, the icon and the index. The Saussurean model of the signifier and the signified referred to them as three modes. The type of mode or sign we are concerned with in this study is the symbol.

2.1 Symbol

A symbol is something which represents an idea, a physical entity or a process distinct from it. The purpose of a symbol is to communicate meaning. For example, a red octagon may be a symbol for "stop". Numerals are symbols for numbers. Personal names are symbols representing individuals. A red rose symbolises love and compassion etc. Pierce observes that a symbol refers to the object that it denotes by virtue of a law, usually an association of general ideas which operates to cause the symbol to be interpreted as referring to that object. It is a mode in which the signifier does not resemble the signified; it is a conventional or arbitrary relationship that exists between the object and the symbol. Language in general, for instance, has these symbols: letters of alphabet, words punctuation marks etc. Symbols are used in all disciplines of learning but we shall limit our study to the disciplines of mathematics and linguistics. In mathematics, symbols are a precise method of giving lengthy instructions related to numbers and logic. Symbols are communication tools that are invented for easy application. They are used to save the need to write long, plain language instructions to describe calculations and other processes. For example, a single symbol of '+' in mathematics stands for the entire process of addition. The plus sign eliminates the need for a long written explanation of what addition means. Weaver (2007) postulates that every meaningful mathematical statement can also be expressed in plain language. He goes further to say that "many plain-language statements of mathematical expressions would fill several pages, while to express them in mathematical notation might take as little as one line". One of the ways to achieve this remarkable compression is to use symbols to stand for statements, instructions and so on." In linguistics also, symbols are shorter ways of writing instructions and are used extensively especially in the area of syntax. There are many symbols used by linguistics and mathematics but the study will examine only the following: the cross/plus (+) symbol, minus (-), the 'x' symbol, the symbols of greater & less than (> <), the superscript symbols (^{1,2,m + etc.}) equivalent sign (=), the geometric structures, shapes and patterns (circles, triangles, rectangles etc.). Our objective is to show their different representations in the different fields where they operate. The work

recognizes the fact that some of the symbols mentioned above are not limited to the fields of mathematics and linguistics only but we limit our study to these two.

3.0. Analysis

3.1 The Cross/Plus Symbol

According to Lynn (2000) a plus is a Latin term meaning "more". The plus sign (+) is a symbol used to represent the notion of positivity as well as the operations of addition. Consider its operation in mathematics as in:

1a. $4 + 1 = 5$ (b). $28 + 5 + 43 = 76$

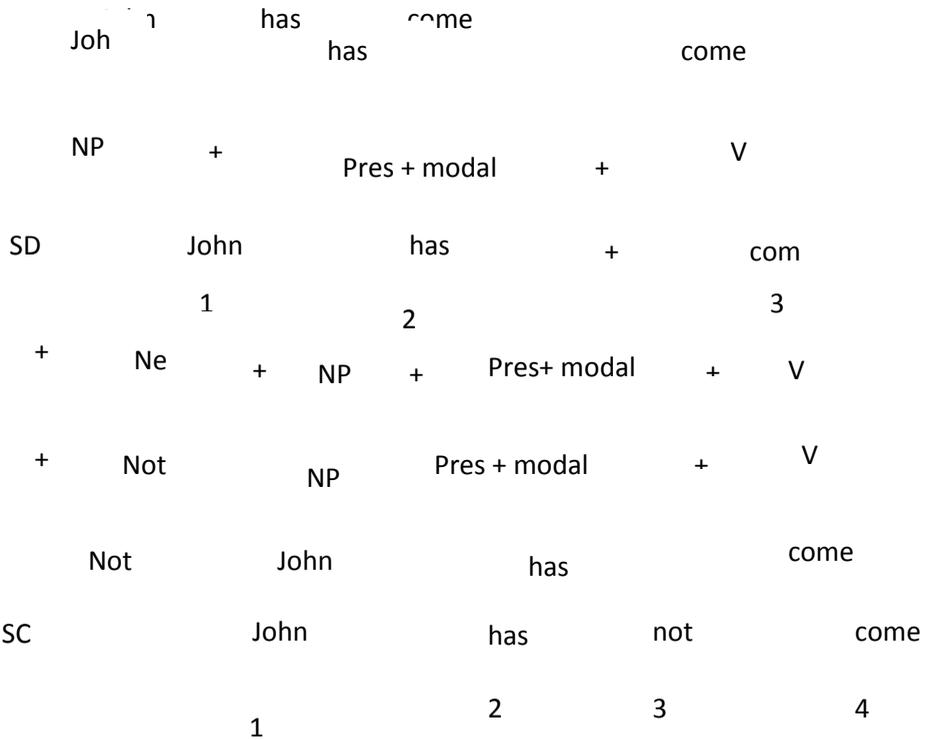
.The use of plus has been extended to many other meanings, more or less analogous. It can also serve as a unary operator that leaves its operand unchanged ($+x$ means the same as x). This notation may be used when it is desired to emphasize the positiveness of a number, especially when contrasting with the negative. Example:

2. $+5, +3, +2$ etc.

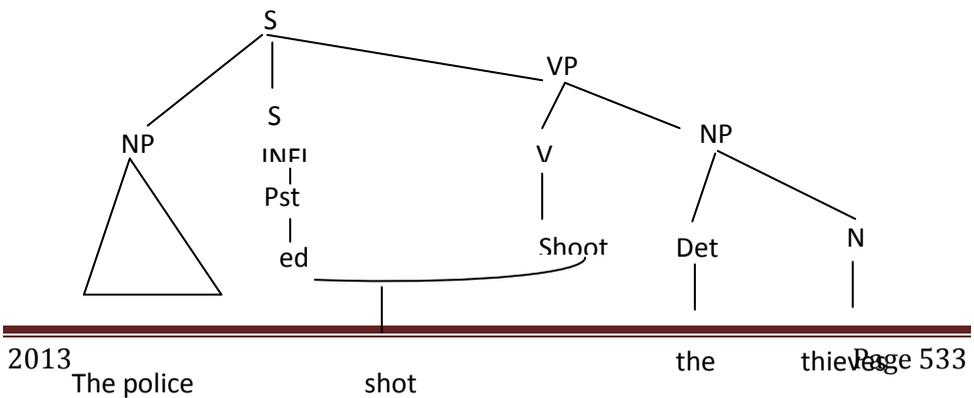
It can also work as a qualifier, for instance, in grading systems (such as examination marks), the plus sign indicates a grade one level higher and the minus sign a grade lower. For example A+ is one grades higher than A. The plus sign can also indicate many other operations, depending on the mathematical system under consideration.

From the data, we observe that a plus sign in mathematics is a tool operator that sums up all the numbers into a unified value. In other words the products are written together, they are not separated. Linguists or syntacticians deriving deep structures to surface structures employ them. Singer (2007) and Delvin (1994) argue that in order to capture abstract patterns and structures of mind and language, linguists find it necessary to use mathematical notations. In linguistics (syntax), plus sign is used in (i) transformations to show the addition (insertion) of a lexical item that was not originally in the kernel sentence. Transformational generative grammar (TGG) is a syntactic model which claims that two levels of syntactic structure exist. The two levels of structural representation are referred to as underlying (deep in the standard theory or D-structure in government and binding theory) and the derived structure. The underlying structure is the structure which has not undergone any grammatical operation, while the output that results from subsequent application of grammatical operation is the surface structure. The syntactic operation which accounts for the relatedness of two superficially different structures is described as T-rule. It selects a sequence of symbols (linguistic, morphemic, word/s and clause/s) as their input and changes them into another set of symbols as the output- (Ndimele (2003) Enonds (1970), Baker (1978), Radford, (1981), Mbah (2011). The Structure which serves as input to transformation is called structural description (SD), index or analysis while the output structure is called structural change (SC). Let us use a T-rule operation of (i) inserting 'NOT' particle into positive sentence to change it to negative sentence. (ii) Movement and insertion – to derive a passive sentence from active one.

(3a)



3b) The police shot the thieves (Active)



The Passive

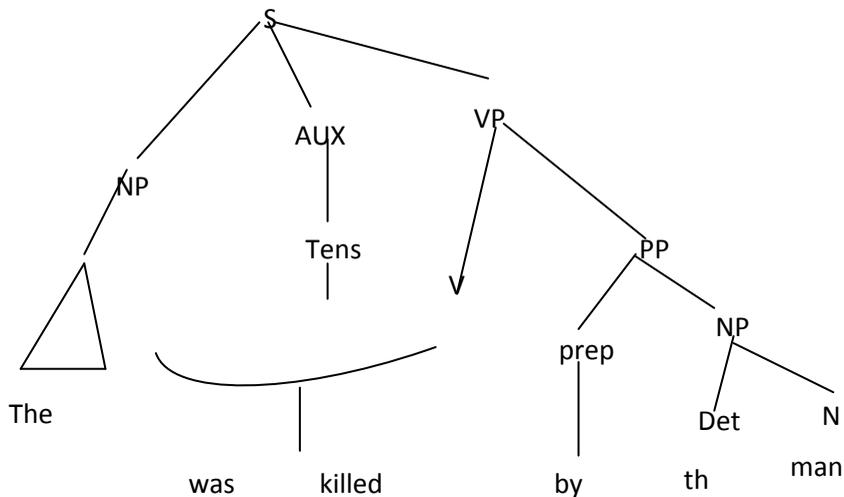
S → NP AUX VP (Adj).

S → NP [Det + N] + AUX [tens pst+ are + pst + s hoot] + VP [V + PP]

Det + N + modal+ pst+ are + VP [tens pst +shoot + PP[prep + Det + NP]

Det + N + modal+ pst+ are + tens pst +shoot + prep + Det + N

The thieves were shot by the police



The data above show that the plus symbol in syntax is a tool employed in transformational operation which allows the addition of a lexical item that is not originally in a construction. This type of addition is referred to as insertion. For instance in '3a', 'Not' is inserted to negate the sentence while 'By' preposition and modal auxiliary (were) are inserted in '3b' to change the construction to passive sentence. Addition in syntax is different from that of mathematics. A plus sign in mathematics yields a product of '2n' from 'n+n' while in linguistics 'N+N gives an output of 'NN'. The summation seems to be the same but, in linguistics, the product results in unlike terms. See below:

4. Mathematics: $n + n = 2n$ or $1 + 1 = 2$

Linguistics: N (Jack) + N (Osmond) → NN (Jack Osmond)

In phonology, a plus sign is used in distinctive feature theory to describe the main class features on the basis of their binary features. The theory analyses segments by assigning them binary values to show that the segment under description by such feature either has

that phonetic property or not. A positive value, [+], shows the presence of a feature, while a negative value, [-], indicates its absence. However, with recent advancement on the distinctive feature theories, scholars of phonology have argued on the existence of single-valued features referred to as univalent or privative features to describe the classes of segments that are said to possess those features, different from those classes that do not have them. For illustrations, we shall use laryngeal features that describes the glottal states of sounds. The examples are culled from Katamba (1989).

5a. \pm (+/-) voice] This feature indicates whether vibration of the vocal cords occurs with the articulation of the segment.

b. \pm (+/-) spread glottis] indicates the aspiration of a segment..

c. \pm (+/-) constricted glottis] indicates the constricted glottis features

Superscripted plus and minus signs are used as diacritics in the International Phonetic Alphabets (IPA) to indicate advanced or retracted articulations of speech sounds. We will not be able to exhaust the use of the ' \pm ' binary values by the IPA with regards to the theory of distinctive features and diacritics due to want of space and the nature of study. In X-bar model, superscripts are used to show the different level projection categories such as: the maximal, the intermediate and the word level. Example:

6. N \longrightarrow N^1, N^{11}, N^{111}

3.2 Negative (Minus) symbol

A minus sign is the inverse of a plus sign. The minus sign (-) is used to represent the notions of negativity as well as the operations of subtraction. Their use has been extended to many other meanings, The minus sign has three main uses in mathematics:

7a. The subtraction operator: A binary operator to indicate the operation of subtraction, as in $4 - 1 = 3$.

b. It means a negative number when it is directly in front of a number and when it is not a subtraction operator. For instance -5 is minus 5.

c. A unary operator that acts as an instruction to replace the operand by its opposite. For example, if x is -3 , then x is -3 , but if x is -3 , then $-x$ is 3. Similarly, $-(-2)$ is equal to 2. In syntax, a minus sign is used in transformation to remove an element in the deep structure to cause it not to appear in the surface structure. This T-rule is called deletion rule. For instance, to return the passive sentence in '3b' to active one as in '3a', a minus (deletion) operator will be used:

8. - (modal) - pst - are (were) and -by (preposition)

We have drawn attention to the use of minus sign in phonology while discussing the use of plus. See example '5' above. A plus/minus symbol can also be used in linguistics for the purpose of grading such as A^+ , B^- etc

3.3 'X' Sign

In mathematics, it is called a multiplication sign. Yoshida (2009) postulates that the 'x' symbol is the mathematical operation of scaling one number by another. It is one of the four basic operations in elementary arithmetic; othes include addition, subtraction and

division. It can be thought of as consisting of some number of copies of the original, where whole-number products can be computed by repeated addition; for example, 2 multiplied by 3 (often said as "2 times 3") can be calculated by adding 3 copies of 2 together:

9. $2 \times 3 = 3 + 3 = 6$. Here 2 and 3 are the "factors" and 6 is the "product".

It is also used to denote, the cross product of two vectors and the cartesian of two sets

The field of linguistics (syntax) uses the 'x' sign in the application of government and binding theory, the principle of government is stated thus:

10. X governs Y if X is the minimal potential
Governor commanding Y, and there is no
Intervening S-bar or NP barrier between X and Y.

'X' is one of the arbitrary symbols used in syntax. In X-bar convention, X stands for any lexical category symbols, Jackendoff (1977) notes that the value of X ranges over at least the lexical categories of N, A, V, P etc.

In projection, Mbah (2003) uses the symbol to discuss the kind of discrete mathematics whose application can be explained by syntactic principles of contemporary syntactic theory. He notes that in phrasal projections...when isomorphic entities are juxtaposed, a path underlies them. These paths enable strings to contract syntagmatic relationship, and produce meanings. The scholar's claim is demonstrated with the example below:-

11. $X_1 X_2 \dots X_N$ (where $N \geq 1$ and (where X is a lexical)
 $X_1 \rightarrow X_1 \rightarrow X_1 + /$ is an arc because for each $1, X \leq I < X$.
If a non – lexical construct is made up of isomorphic entities
 X^1 to X^{maximum} , then X, is an arc, which links the first
lexical items to the order, to X^{maximum} or the phrasal
projection,
for instance, if X_n is made ulo, ocha at\o 'three white
house' the ulo includes an arc or an agreement feature
which enables it to grammatically relate to X^1 , which is
' ocha ' and so on. X_n is greater than. It implies that ' ulo
 ocha at\o ' is greater than ' ulo ocha ' or ' ulo ' in that order
or X_n is equal to I if the whole phrase is regarded as an
entity-Mbah (2003:81)

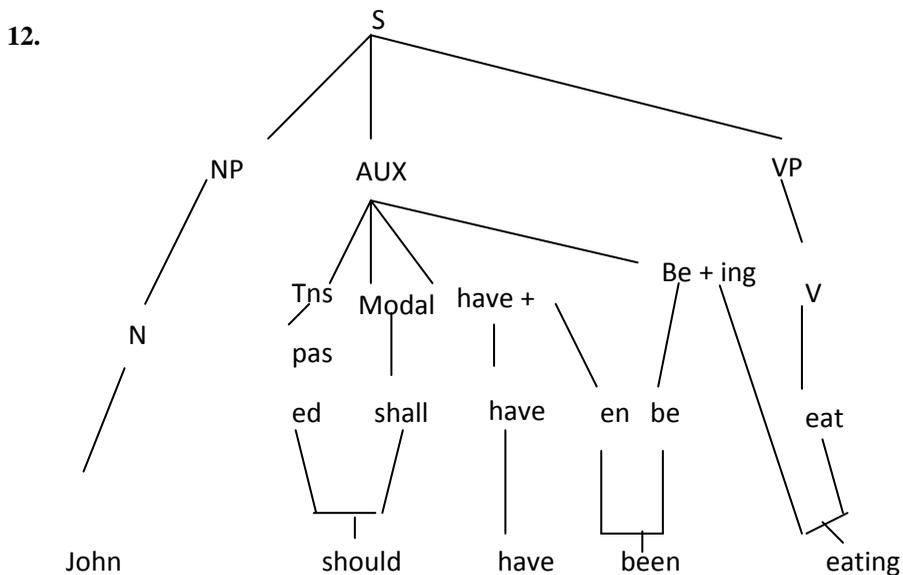
The illustrations above shows that the X notation is a feature of syntactic derivation and projections. Linguistics uses the symbol as a strong operator in stating some syntactic equations.

The Use of Arrangements, Patterns, Structures and Shapes

Yiparaki (1999) notes "unlike formalism structuralism allows mathematics a subject matter; it grants mathematics unlimited generality and applicability". Pepin

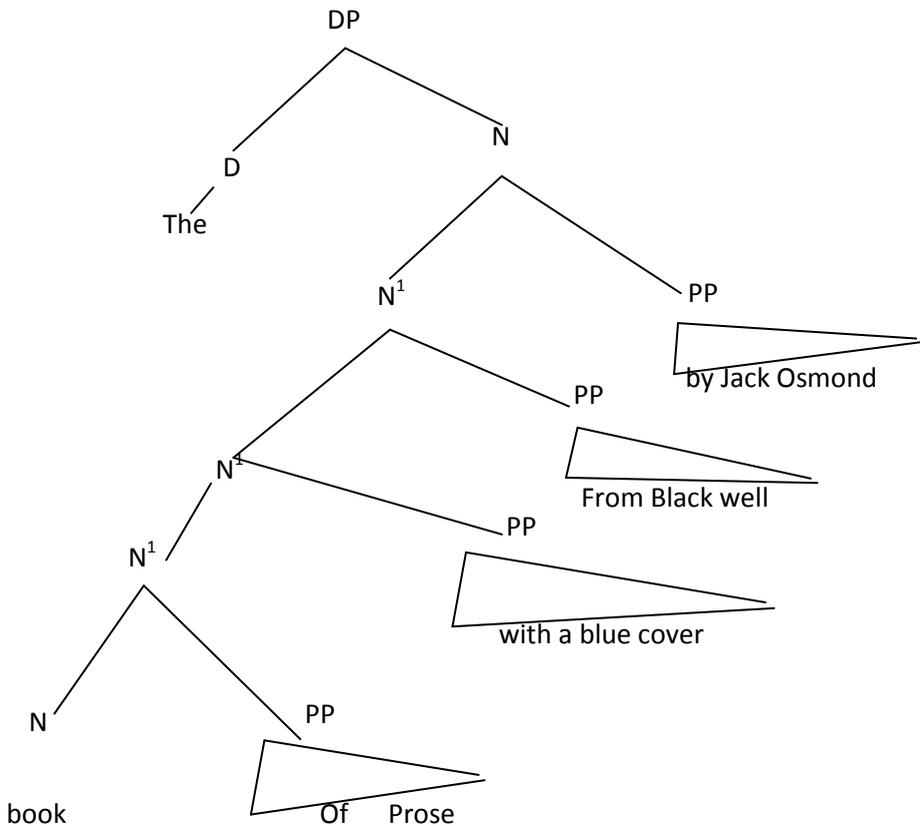
(2007) defines mathematics as a science dealing with the logic of shape and arrangement. Gold (2007) postulates that mathematics is concerned with the property of patterns. Viewing mathematics as a property or science of patterns implies a structuralist perspective. Agreeing with Gold, Ongstad (2007) claims mathematics is conceived as a 'sky-scraper' rather than as a row of traced houses. From the views of the above scholars, mathematics uses structures, shapes etc. for actualisation of its content. Such shapes include: circles, triangles, rectangles etc. Mathematicians work on circles to find the area, the radius, circumference using 'pi' symbol. In triangles, the angle of elevation or depression etc. is the focus for mathematicians. Syntacticians apply structures to phrases, clauses and sentences. In syntax there is a theory of sentence structures and in generative grammar, Lemke observes that language was taken further in structural direction by Chomsky's generative approach; generalisations about structure are represented by rules. The rules are said to "generate" the tree, though trees may be generated in a particular way using a particular rule. There are such rules as phrase structure, X bar, T – rules etc. in which case T – rules generates transformational structure tree of a sentence. A syntactic sentence in a tree structure is popularly referred to as a tree diagram. According to Ndimele (2003), it is a grammatical device which is used to show the configuration of a construction. It specifies the hierarchical structure of linguistic unit at the various stages of derivation. The whole essence of this tree is to show structural relationship between the constituents that make up a construction.

With the use of PS rules and T – rules let us show the relationship of the elements in the construction below:

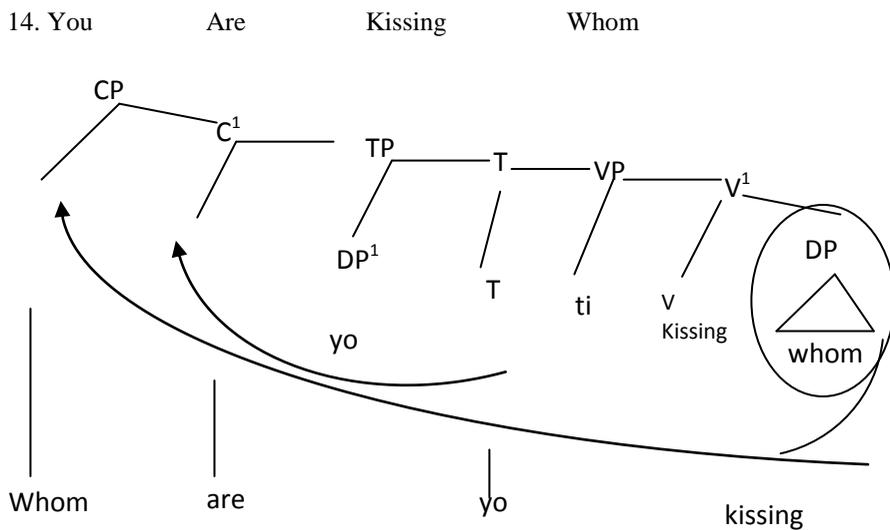


In geometry, mathematics uses shapes and patterns a lot. The discipline, syntax, also employs these tools in structural descriptions. Carnie (2007:102) opines that “a collection of lines and labels with an internal organization, like syntactic trees is a geometric object”. It is necessary to concern ourselves with the mathematics of tree diagrams and other shapes / patterns so that we can assign names or labels to show their relatedness. Looking back at the frame of (3b) a shape of a triangle (∇) was used under the NP node. The node has two lexical items under it, the triangular shape there accounts for the possible node of housing two words from different categories. In other words, shapes and patterns are used in linguistics to house a phrase by a node, to show movement and their traces etc. consider the structure below:-

13. The book of Rose with a blue cover from Blackwell by Jack Osmond.

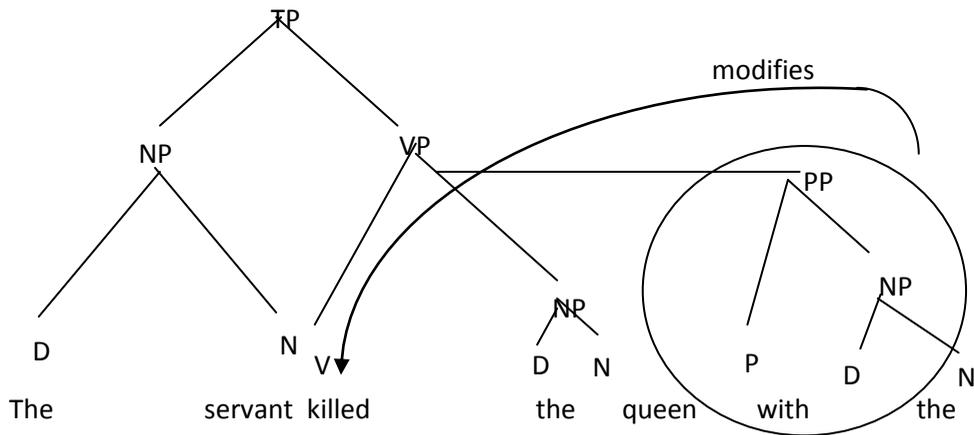


The structure above shows that the PP nodes host at least two constituents which are of different word classes each. This is usually aimed to gain space and make the structure appear smaller though at times it is a style of an author.



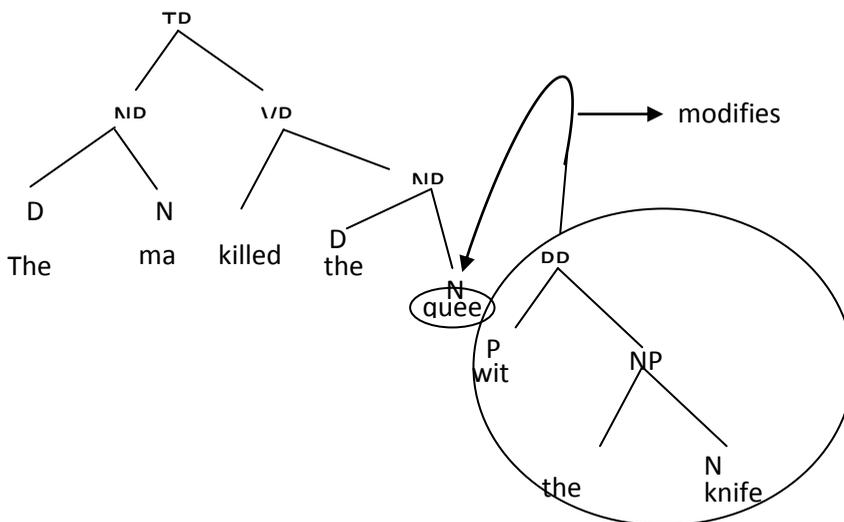
Structure ‘14’ shows a highlight of the moved constituent “Whom” to the complement (comp). The moved constituent is circled for easy identification. Partee (1990) discusses the principles of modification. He uses circles to distinguish modifiers from their heads in an ambiguous construction. For example, “The servant killed the queen with the gun”. To show that this sentence has two meanings, he applies these principles of modification – If an XP modifies some head Y, (i.e. a daughter of YP) in a construction with the tense

phrase (TP) above, the circles shows which constituent or phrase modifies which. First Meaning – The servant used a gun to kill the queen.



In the structure below the PP (with the knife) modifies the verb (killed) The PP here describes how the queen was killed.

(Second) Meaning – The queen with the gun was killed. In the instance, the ‘PP’ with the gun modifies the queen and therefore attached to the NP as circled below 5h).



The illustrations in 'Sg' and 'h' shows that syntactic trees helps to capture the differences between ambiguous readings of the same surface sentence while the shapes of circles are used differentiated modifiers from heads.

Summary of the Findings and Conclusion.

The work studied some of the similar symbolic notations applied in Mathematics and linguistics to ascertain the different representations of the same notations. The two disciplines make use of the following symbols among others: the positive/cross notation (+), negative/minus sign (-), the 'x' sign, the symbols of inequality $>$ $<$ (greater & less than), the superscript symbol, the equivalent/equal sign (=), structures, shapes etc. Our findings are that the positive/plus sign in mathematics represents addition while in linguistics (syntactic transformation), it is an insertion of word/words in a phrase/sentence whose output cannot be written together. In phonology, a plus sign shows the presence of class features. In mathematics, the superscripts are used to show the power to which a number can be raised. In linguistics, they represent the levels of phrasal categories (the maximal, the intermediate and the lexical). The multiplication sign is used for scaling operation in maths while in linguistics, it stands for any lexical category symbol (N, V, A, P etc.). It is also used in projection operation. The work therefore, discovers that most of the symbols though similar have different representations in the two disciplines.

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