Uncertain Information: A justification for Alternatives to Classical Logic

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Abstract
The main objective of this paper is to argue that uncertain information is a justification for the creation of systems of alternatives to classical logic. Uncertain information is information that some of its necessary data is not available as at the time it is needed.Uncertain information is necessary, especially for growth and development of organisations. Classical logic, the traditional form and most accepted form of logic in Philosophy of Logic, is considered by scholars such as James Tomberlin as the only justifiable form of logic and considered other systems of logic unjustifiable and unecessary. This is without considering the limitations of classical logic. This paper argued that although classical logic is a valid logic in its own right, it is limited in handling uncertain information, but systems of non-classical logic, which are alternatives to classical logic, handles uncertain information appropriately. The study employed the methods of logic. These included conceptual analysis and critical analysis. Relevant literatures on logic and computer science were consulted. The literatures used were drawn from classical and contemporary texts. The paper also used the method of philosophical argumentation to show that the use of alternatives to classical logic in handling uncertain information serves as a justification of systems of non-classical logic in Philosophy of Logic. The first finding of the paper is that alternatives to classical logic are possible. Second, the paper established that there is no contradiction between classical logic and systems of non-classical logic. Third, it was established that classical logic is valid but is limited in handling uncertain information. Fourth, it was argued that a persuasive justification of systems of non-classical logic is their ability to successfully handle uncertain information. The paper concluded that systems of non-classical logic are valid just as classical logic. A major justification of systems of non-classical logic lies in its application in computer science, especially in handling uncertain information. Consequently it follows that systems of non-classical logic works where information is uncertain.

Key words: Classical Logic, Non-Classical Logic, Certain Information, Uncertain Information.

Introduction

Certain information is information that contains all the necessary data. For example “It is raining now”. Uncertain information is information that lacks some necessary data as at the time the information is needed.[i] For example "There will be a battle in Warsaw next year". As at this time there is no sufficient data to ascertain if the information in this statement is true or false. However, the information is still crucial. Information, whether certain or uncertain, is crucial to the growth, development and well-being of any organisation or even Nation. It is therefore important that systems of logic can adequately handle both certain and uncertain information. For example, workers in a stadium with seating capacity of two thousand fans have to prepare for a match without incurring a loss, even though they are not sure of the number of fans that will turn up for the match. Statements convey information and Logic is the science that evaluates statements; hence, it is important to know how logic can adequately handle information like this.

The traditional form of logic is classical logic. Classical logic is based on the principle of bivalence. The principle states that every statement is necessarily either true or false. For example, it is necessarily true or false that "It is raining now". Classical logic allows for two possible truth-values: 'true' and 'false'.However, Aristotle argued that when the principle of bivalence is applied to contingent statements (statements referring to future events), for example "There will be a battle in Warsaw tomorrow"[ii] we have to accept a fatalistic conclusion. Though, Aristotle never created an alternative to classical logic, his essay provoked some philosophers into creating alternatives to classical logic. These alternatives are known as systems of non-classical logic and are supposed to accommodate more truth-values than true and false.

There are arguments for and against the possibility and justification of alternatives to these systems of classical logic. Such arguments include the argument from contingent statements, the argument from truth, the argument from the implication of non-classical logic, the argument from the premise of non-classical logic and the pragmatic justification of non-classical logic. James Tomberlin argue that systems of non-classical logic are unacceptable because they are not necessary, not justifiable and inconsistent with the principles of logic.[iii] This paper will argue that although classical logic is valid in its own right, it is inadequate in handling uncertain information, which systems of non-classical logic can handle appropriately and this serves as a major justification for systems of non-classical logic.

The paper will first examine some of the arguments that systems of non-classical logic is unjustifiable. The paper will then establish the relationship between Philosophy of Logic, Computer Science and Artificial Intelligence. The paper will then identify types of information and their importance. The paper will then show why classical logic cannot handle uncertain information and can only manage certain information. The paper will conclude by assessing the success of systems of non-classical logic in handling uncertain information and establish that this is sufficient justification for systems of non-classical logic.

Classical Logic versus Non-Classical Logic

According to James Tomberlin, although systems of non-classical logic are interesting, there is absolutely no need to create them, since there is no problem their creation will address.[iv] It was the urge to solve the problem of fatalism, Tomberlin argued, that led to Jan Lukasiewicz's creation of a system of non-classical logic. However, since the fatalistic conclusion cannot be justifiably inferred from Aristotle's argument, without committing a fallacy, there is no need to delve into the realm of any system of non-classical logic.[v]

We can infer the following from Tomberlin's arguments: deducing a fatalistic conclusion from Aristotle's sea paradox is a fallacy; this fallacy has led to the misconception that we cannot determine the truth value of contingent statements without drawing a fatalistic conclusion; and this misconception has led to some complicated "solutions", one of which is the creation of systems of non-classical logic. The realisation that the inference is a fallacy puts to rest all these complicated solutions, because they are unnecessary. If fatalism cannot be inferred from the principle of bivalence, then is there any need to create systems of non-classical logic?

Another interesting argument in the controversy between classical logic and non-classical logic logic is the argument that Lukasiewicz assumed erroneously that factual truth is identical with necessity and factual falsehood is identical with impossibility.[vi] According to Charles Baylis and **A.P**. Ushenko, who subscribe to this argument, when we say that 'something' is possible we mean that that thing is conceivable. For example, if I say that it is true that Mr. Jan will go to Warsaw next year, all I am saying is that there is no logical contradiction in the assertion that "Mr. Jan will go to Warsaw next year". In actual fact, Mr. Jan may not go to Warsaw next year, but there is a possible world where it is true that Mr. Jan will go to Warsaw next year.[vii]

Conceivability in logic is not a report of some mental act, but the ability to create worlds about which we can make assertions without contradiction.[viii] That is, an event can be conceived (logically) as true or false, even before its occurrence. The mere fact that I say "It is true that there will be a sea battle tomorrow" does not imply that there will be a sea battle tomorrow, and the fact that I say "There will be no sea battle tomorrow" does not imply that it is impossible for a sea battle to take place the next day. Truth is different from necessity and falsehood from impossibility. If this is the case, then even if it is true that a sea battle will take place tomorrow, it does not follow that it must necessarily take place tomorrow. Though, it is absolutely possible that it will take place and absolutely possible that it will not take place.

Another challenge that systems of non-classical logic will face, as argued by Ushenko, is that it does not make sense to verify the "authenticity" of a statement that has 'possible' or 'doubtful' as its truth-value.[ix] When a statement is false, the conditions under which we establish the statement as false can be verified. And when a statement is established as true, the conditions under which we establish the statement as true can be verified. However, when a statement is established as 'doubtful' or 'possible', there are no such conditions.

A likely defense for systems of non-classical logic is that just as a statement has conditions under which it can be established as true or false, so also does a statement established as 'doubtful' or 'possible'. If we have to examine these conditions to verify if a statement is actually true or actually false, then we can as well examine the stated conditions under which a statement can be established as 'doubtful' or 'possible'.

Ushenko seems to lack an understanding of what the notion of truth is in logic. Truth in logic is a relation between words and not between facts as suggested by Ushenko. Ushenko did not explain what he meant by "verification". If by "verification" he meant a correspondence between facts, then his notion of truth is not the notion relevant to logic. Furthermore, there seems to be no problems with giving conditions for why a statement is established as ‘doubtful’ or ‘possible’. One possible condition for establishing that the value of a statement is ‘doubtful’ or ‘possible’, is that if we cannot establish whether the statement is ‘true’ or ‘false’, we have established that the statement is ‘doubtful’. The problem is not with laying down conditions under which a statement can be established as ‘doubtful’ or ‘possible’, the problem is whether the conditions are acceptable or not.

Another defense for systems of non-classical logic lies in what it can do. Is there anything that classical logic cannot do that systems of non-classical logic can do? Rather than just discard many-valued logic, I will like to examine whether there is such value of many-valued logic that can make it worth retaining. If we can get any, then it may be that the systems are after all an advantage and not a problem in itself.

I argued earlier that there is nothing about contingent statements or statements that have vague objects and concepts within them to suggest that they are not capable of being either true or false. I argued that the challenges we face in determining whether the truth-value of these statements is more of an epistemic challenge. However, when we need to use these statements in formal reasoning, how do we handle them, despite the epistemic problems?

We have to examine three things; first, we have to determine whether there is any need for these statements; second, we have to see whether classical logic can handle them adequately, and third, if classical logic cannot handle them we have to see whether non-classical logic can handle them better. If non-classical logic can handle them adequately or better than classical logic, then there might be a good justification for systems of non-classical logic from uncertain information.

Logic and Computer Science

Computer science is the study of the theoretical foundations of information and computation, their implementation and application in computer systems development.[x]Hence, methods for understanding, formalising and modelling techniques or applications for management of information are a vital part of the computer sciences. Artificial intelligence is the aspect of computer science that is concerned with building models or machines that can assume some behaviour considered as intelligent.[xi] When data is processed into a machine it produces information. It is the data fed into the machine that determines the information that will be produced. Human beings can process data, but it takes a shorter time when the data is imputed into a programmed machine and processed into useful information.

There are two types of information; certain information and uncertain information.Certain information is information in which all the data is available, while uncertain information is information in which some of the necessary data is not available at the time the information is needed. However, uncertain information is essential to the growth of any organisation. When information is incomplete, inconsistent, ambiguous, inexact or imprecise it is called uncertain information.[xii] There are some assumptions relied on in artificial intelligence. These assumptions include the following:

1. That information can be composed. In other words, information can be created.
2. Information can be correct according to some measures and also incorrect according to some other measures.

3. We have expectations about information. For example, the availability of certain information.[xiii]

There are several definitions for logic. It is,however, generally understood as the discipline or science that evaluates arguments and the principles of valid inferences and demonstration. According to **William Kneale and Martha Kneale**, "Logic is concerned with the principles of valid inferences."[xiv] According to Wilfrd Hodges beliefs are expressions through written sentences.[xv] According to **Copi**…… Hence, statements convey valid inferences, arguments and information. Logic is the science that evaluates these statements. Logic studies and represents the features of statements and inquiries into the nature of the features common to statements and the relations between them.[xvi]

Furthermore, Kant described logic as the science of judgement.[xvii] Kant saw judgement as a function of reasoning. He defined judgements as complex conscious cognitions. Logic is the science that displays the possible forms through which judgements are made, and the structure of judgements.[xviii] We can understand Kant's conception of logic as the science that studies the patterns of formal reasoning. It seems that all the definitions and characterisationof logic shows that logic is the science that evaluates the statements that convey information. This highlights the importance of logic in evaluating information.

Artificial Intelligence

Artificial intelligence is the aspect of computer science and engineering that studies and develops programs that enable computers to exhibit intelligent behaviour.[xix]. According to John McCarthy, who coined the term 'artificial intelligence', "Artificial intelligence is the science and engineering of intelligent machines."[xx] The functions of systems created by artificial intelligence include ability to set goals and achieve them, to visualise, predict and represent the state of the world, represent knowledge, plan and determine the value of choices available to it. One of the major tools used by artificial intelligence to achieve these functions in computers is logic.[xxi]

Some of the problems in artificial intelligence are: representing knowledge and finding the essence of abstract reasoning. According to John McCarthy, there is a need to focus on logic to solve these problems.[xxii] For Robert Moore, logic plays a crucial role in some core areas of artificial intelligence especially as a tool of analysis, basis for knowledge representation and programming language.[xxiii] Many problems in artificial intelligence studies require working with incomplete or uncertain information. When artificial intelligence has to represent knowledge using incomplete information, it sometimes has to assume that almost no statement is simply true or false in the way represented by classical logic.[xxiv]

The forms of logic used in artificial intelligence includes classical logic, fuzzy logic and default logic.[xxv] The importance of artificial intelligence to any serious minded organisation cannot be undermined. Artificial intelligence is used to research on medical diagnosis, stock trading, law and scientific discoveries.

Uncertain Information

It is argued that no organisation can survive on certain information only;[xxvi] hence, there is a need to learn how to manage uncertainty in information systems. For example, only uncertain information can be used to predict the future performance of investments, to identify market behaviour, to design products that will meet the client's specifications, to write new articles, to determine if an insurance claim should be accepted or rejected or to provide legal advice. These can only be done with uncertain information.

To achieve these goals, an organisation must make use of information from different sources and this information will largely be uncertain or incomplete. Using uncertain or incomplete information involves a lot of risk, since not all the data needed are not immediately available. However, since using uncertain information is essential to the growth of any organisation. The rationale for using uncertain information is that the risk and possible mistakes that can be incurred in using uncertain information should be reduced to the barest minimum.[xxvii]

Since handling of uncertain information is delegated to computers, the principles involved in handling this information should be well understood. Unless the principles are well understood, the behaviour or result of the database can be counter-productive. Artificial intelligence, therefore, seeks a reliable and fast model, which is simple, yet efficient, for handling uncertain information. Uncertain information is presently grouped into four classes; probabilistic information, fuzzy information, inconsistent information and default information.[xxviii] The type of uncertain information determines the model that can be used to handle it.

Types of Uncertain Information: Probabilistic, Fuzzy and Inconsistent Information

Probabilistic information is acquired by predicting values through different strategies, for example, betting. Most probabilistic information is based on subjective factors; hence, it cannot all be immediately identified as strictly true or false. Fuzzy information is information that is vague or not clear enough. However it is useful. For example, in respect of information on fast cars and big houses, what will fit into the class of fast cars or tall houses will most likely be vague.

Inconsistent information is information which is used together with its contrary. It usually occurs when information is collected from many sources or people. For example, through interviews and administering questionnaires. It might be undesirable, but can be useful. When an organisation strives for absolute consistency, this can lead to a huge loss of important information. Hence, it is sometimes better to accommodate different views together and analyse them by sound reasoning, to determine which should be discarded and which is useful.

Default information is hardly accurate; however, without recourse to default information everyday life will almost be impossible. For example, when going to work, school, another state or nation, we assume these places are still in the same place they used to be and that we still have a place there. Without these assumptions, which is default, why go to these places? Recourse to defaults allows us to make conclusions that are useful even when the available information is not sufficient.

Classical Logic as an Approach to Handling Uncertain Information

Well understood principles are required when the handling of uncertain information becomes necessary; otherwise the result will be counterproductive. A range of techniques are developed and a reasoning component is incorporated into the information system to get productive results from uncertain information.[xxix]

One of the major approaches to the handling of uncertain information is classical formalism. Classical formalism includes classical sentential logic, classical predicate logic and classical set theory.[xxx] There are some programming languages and database systems based on classical logic. A range of databases for propositional and predicate logic and the consistency of a set of statements can be automated.[xxxi] A database based on classical logic can be used to handle some forms of uncertain information. For example, if we have the information that a woman just gave birth to a baby but we are not sure of the baby's sex, what we have is incomplete information. We can use the disjunction to derive the information that "The woman gave birth to a boy or to a girl." This statement is true, though we are not sure of the actual gender of the baby.

However, uncertain information that is probabilistic, fuzzy or/and default cannot be properly handled by classical logic. Classical logic relies on the principle that all statements are either true or false. However, uncertain information deals with statements that are sometimes neither true nor false, both true and false, not completely true, almost true, not totally false and so on. Hence, there is a need to have alternative ways for handling uncertain information. Uncertain information is important for the growth of any organisation. This necessitates the pursuit of other reasoning systems that can handle uncertain information. A database or programming language based on classical logic will be too weak to handle some uncertain information.

Alternatives to Classical Logic Based Database

When Aristotle wrote his essay on future contingents, it served as an eye-opener to some philosophers, for example, Lukasiewicz. These philosophers had to seek, outside classical logic, approaches that could handle contingent statements, for example, systems of three-valued logic, since contingent statements are meaningful statements and cannot just be discarded. In computer science also, there is uncertain information that cannot be handled by classical logic. There are instances where the available information to reason with can be unclear, vague, inconsistent, incomplete or fuzzy. This requires applying a system of many valued logic. So, uncertain information is examined and a more adequate approach to handling them is presented.

Handling Fuzzy Information

Most of the concepts we use in human reasoning are fuzzy, unclear or vague, but useful. Sometimes, precision in information can make accomplishing some tasks difficult. For example, it is easier to park a car when there is no precise specification on how it should be parked. Classical logic cannot handle fuzzy concepts, for example, classical logic can handle the concepts such as men, houses, trees, but cannot handle classes of fat men, big trees, large houses, because they are vague classes. Hence, we need an alternative to classical logic to handle fuzzy concepts.

One area in which fuzzy information is largely in use is in solving engineering problems. This is because fuzzy systems are faster in modelling systems. Fuzzy set theory is an intuitive and useful extension of classical set theory for reasoning with fuzzy information. There are some applications where fuzzy information is being used in reasoning, for instance, fuzzy control systems and fuzzy expert systems. Fuzzy set theory and fuzzy logic are some of the approaches towards formalising reasoning with fuzzy information.[xxxii]

Fuzzy Logic

Fuzzy logic is a logic that has been extended to handle the concept of partial truth- truth values between "completely true" and "completely false".[xxxiii] In fuzzy logic, truth values are extended from the classical true and false truth values to the following truth values; true, more or less true, rather true, not very true, not very false… false.[xxxiv] Truth values in fuzzy logic are defined as a function of each other. That is, when we have the value for true, we can define the value for some other truth value. For example, in classical logic, the value 1 denotes true and the value 0 denotes false. However, fuzzy logic assumes that between 1 and 0, there are other values.

Some of the other values between the traditional true and false, in fuzzy logic, include; '*not very true*', *'not very false'*, *'rather true'*, *'more or less true'* and *'more or less false'*. Fuzzy logic is an attempt to incorporate values for handling uncertain and incomplete information into a reasoning system. The assumptions underlying systems of many-valued logic underlie systems of fuzzy logic. For example, a three valued system of fuzzy logic can define its third value as *'unknown'*, *'possible'* or *'undecidable'*. The same definitions for connectives and values in Lukasiewicz's system of three valued logic will be applicable when the third value in the system of three valued logic is *possible*.[xxxv] A typical example of a system of three valued system of fuzzy logic is Lukasiewicz's system of three valued logic.[xxxvi]

The definitions for a system of four valued fuzzy logic usually overlap with the system of four valued logic in many valued logic.[xxxvii]Any definition for a function value will produce different types of fuzzy logic. Hence, the class of fuzzy logic provides a wide scope of approaches for reasoning with fuzzy information. Fuzzy logic can range from finite to infinite valued logic system. Fuzzy logic is a good example of an infinite-valued logics system; it is a way of increasing the number of truth values from two to three, four or more truth values. In general, the intuitions behind the truth values for many-valued logics correspond with the truth values for fuzzy logic.[xxxviii] Fuzzy logic is not only used to analyse fuzzy or unclear information, it can also be used when the available information is inconsistent.

Handling Default Information

Default information is used when there is no alternative information available. Default information can either be non-defeasible or defeasible. Non-defeasible information is usually conclusive and hardly requires any adjustment. For example 'Water boils at 100^0C' is considered non-defeasible information, despite the fact that we can never test the absolute truth of the information or statement. Defeasible information may require amendment, at one point or the other, in data processing. An example of defeasible information is 'Matches light when struck'. However, the information may change when there are instances of matches not lighting even when struck.

Working with default information, especially defeasible information, sometimes requires withdrawing an inference already made, in case more information becomes available. Defeasible information allows us to assume that whatever has not been found false in a set of data is true. This assumption allows us to proceed with storing and processing information for maximum benefits. If we have to wait for every information in a set of data to be proven true or false, we may never be able to maximise the use of that information, especially when it expedient we do so.

Handling default information, especially defeasible information, requires careful reasoning and analysis; we cannot just remove or add information arbitrarily. Non-monotonic reasoning, for example, default logic, is usually used to handle default information in artificial intelligence. Non-monotonic reasoning is used in conjectures; that is, it allows us to infer that any given Ø is Ψ, if we have the knowledge that most Ø are Ψ. It is also used in learning. If a general or inductive inference is made and an exception to that inference is later discovered, non-monotonic reasoning allows us to handle the exception without having to discard the whole inference learnt. Otherwise, a lot of things will have to be learnt from the scratch if we have to discard an inference learnt anytime an exception to the inference emerges.

One likely problem that may be encountered with applying the assumption that any given Ø is Ψ, if we have the knowledge that most Ø are Ψ is the problem of hasty inductive generalisation.[xxxix]Default logic is supposed to justify the generalisation made over the information so as to avoid the problem of hasty generalisation. Default logic will not only assume that it is sufficient to generalise that any given Ø is Ψ, if we have the knowledge that most Ø are Ψ. It will have to incorporate within the system other arguments for making the generalisation. For example, it may need to show the expedience of making the generalisation and the likely problems that may be encountered if recourse is not made to a default.

Classical logic can handle non-defeasible information, since the information needed is usually based on statements that are either true or false. However, classical logic will not always be able to handle default information, especially when it is defeasible. Handling defeasible information requires recourse to non-classical systems of logic for formalism of information. One of the advantages of recourse to defaults is that it reduces the size of database. When the size of database is reduced, it is easier to handle the information in it.

Handling Inconsistent Information

Inconsistent information is information that co-exists with its contrary. Reasoning has different perspectives to it and inconsistencies are a perspective of reasoning. Maintaining absolute consistency in reasoning at all times may not only be possible, but also undesirable at times. Absolute consistency may lead to loss of important information in data and even some profit in business.[xl] Moreover, the actual world forces us to rely on inconsistencies several times. Rather than doing away with inconsistencies, we have to learn how to handle them.

Consider this example: A cinema house that has a 1,000 sitting capacity sells 1,100 tickets a week to a show. This is inconsistent with the number of seats available; however, the managers of the cinema house will maintain this inconsistency because they do not expect everyone who bought a ticket to actually attend the show. Hence, at the time of the show, it is likely that the inconsistency will resolve itself and the cinema house can make extra money. There is, therefore, a cost-benefit in maintaining inconsistencies. The goal of most organisations is to make profit. An organisation must be able to reason on how to maximise profit, even when it requires projecting into the future and handling contrary information.

As useful as inconsistent information is it is not desirable when not handled properly. Normally classical logic does not support contradictions in inferences or propositions, for instance, Ø, ~Ø in classical logic is dismissed as bad reasoning. There is yet no perfect approach to handling inconsistencies, but an effective formal approach is to recourse to four-valued systems of many-valued logics.

Conclusion

This paper attempts a justification of alternatives to classical logic using their success in computer science and handling of uncertain information. The use of many-valued logic in computer science, especially artificial intelligence, to create software and programmes that can maximise the use of uncertain, inconsistent and incomplete information is discussed. Human beings thrive on degrees and variations. Between 'yes' and 'no', 'true' and 'false' there are diverse degrees. Handling these degrees and variations in organisation require systems of non-classical or many-valued logic. For example, fuzzy set theory in mathematics and computer science are used to handle degrees and variations in reasoning.

The paper concludes that although classical logic is a logic in its own right and alternatives to classical logic do not entail its rejection, classical logic cannot adequately handle some necessary and important information. Hence, systems of non-classical logic are not only justifiable, but also necessary to handle uncertain information.

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[i]Mendel, Jerry. *Uncertain Rule-Based Fuzzy Systems.*Gewerbestresse: Springer Pub., 2017. 245.

[ii] The information in this statement is uncertain information. Although it is clear that the statement is either true or false, as at the time the statement is made there is no sufficient evidence that the statement is actually true or the statement is actually false.

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[xxv] Default logic is proposed by Reiter Raymond to formalise default assumptions. There are some things we accept as true by default, despite the fact that they have exceptions, for instance, all birds fly. Default logic allows us to accept that all birds fly is true without stating the exceptions.
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[xxvii]Hunter, Anthony. *Uncertainty in Information Systems*.2.
[xxviii]Hunter, Anthony. *Uncertainty in Information Systems*.5-7.
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