On The Rationality Of The Indubitability Of Cartesian Mathematical Propositions

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Abstract

Descartes was no less hungry for certain and indubitable truths than Plato and other of Descartes' predecessors. This quest for certainty made him design a procedure for the direction of the mind in the process of acquisition of indubitable knowledge. And very curiously, he advocated that truths of philosophy should be approximated after the like of geometric truths. This paper is proposed to investigate under what logical pretexts Descartes could be justified to suppose that geometric truths are indubitably certain. As a result, the conclusion is reached that geometric truths are neither analytically nor existentially indubitable as Descartes had thought. This paper consider the Cartesian procedure for the acquisition of knowledge, considers some mathematical propositions, then reaches the conclusion that it is difficult to find such mathematical propositions which are so certain that they cannot be doubted.

The origin of Cartesian epistemic doubt

The Copernican revolution, which repositioned planetary bodies, was one single epistemic phenomenon which dispirited knowledge searchers. This feat was again one single phenomenon that undermined ecclesiastical authority. The Augustinian divine illumination could after all illuminate wrongly. The geometric attitude to epistemology where knowledge was derived, through a rigorous deductive method, from fixed axioms that were purported to be intuited through "the natural light of reason' was to be treated with disdain. Even though Copernicus first wrote this *revolutionisbus oribium Celestium* anonymously, and Galileo was dragged to the inquisition, but the die was cast and the state was set for a paradigm shift from ecclesiastical domination in matters epistemological. If the geocentric interpretation of planetary and celestial motion was wrongly divinely illuminated, then, by deduction, several other generally accepted beliefs may have been wrongly so generally accepted. Ecclesiastical authority was thus heavily diminished.

Knowledge is like a yearning which cannot be satisfied by arbitrary postulations. Once there is an inner conviction that there is a logical imbalance in any epistemological representation, cognition stands so vehemently distorted and there will be no inner tranquility. Nicholas Copernicus was a catholic, infact, a Jesuit priest. He did not find the church's approval of geocentricism to be in consonance with empirically observable phenomena. His religious status not withstanding, he defied the authority of the church. Several years later, and with the invention of the telescope by Galileo in 1510 he (Galileo) corrobated Copernicus' heliocentric hypothesis. The wrath of the church was there everywhere and staring them in the face. But there was an inner conviction. And their conviction was vented.

The fallacy of *ad vericundiam* formed the basis for Francis Bacon's idols of the theatre. These idols need be pursued away while we return back to things themselves for an understanding of their nature. Descartes, as a matter of fact, went beyond this phenomenalism. He doubted even the existence of things themselves. Curiously however, mathematical truths for him were sacrosanct and he regretted why (philosophical) knowledge was not built upon such “stable foundation” (82).

The certainty of Geometric truths

Because of the flooding of the banks of the Nile (the longest river in Africa), it became expedient for the Egyptians to redefine the boundaries of various persons’ parcels of land. Furthermore, the Egyptians pyramids required measurements of their bases, heights, etc. Documented evidence of practical geometry reveals that it started in Egypt. It is also worthy of note that ancient Egyptians moved from measurements of lines and distances to numerical calculations (i.e. arithmetic). One of the texts popular as a copy exercise in the Egyptian mysteries school as a satirical letter in which one scribe, Hori, jettisons his opponent, Amenme-Opet, for his incompetence as an adviser and manager. It reads:

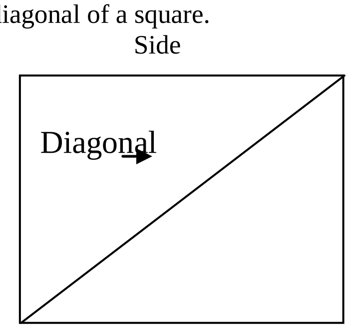
> You are the clever scribe at the head of the troops. A ramp is to be built, 730 cubits long, 55 cubits wide, with 120 compartments. It is 60 cubits high, 30 cubits in the middle… and the generals and the scribes turn to you and say. “You are a clever scribe, you name in famous. Is there anything you don’t know? Answer us, how many bricks are needed?” Let each compartment be 30 cubits by 2 cubits (Encyclopedia, 577).

Evidently, the ancient Egyptians were as preoccupied with arithmetic as they were with geometry. However, the mathematical operations of addition, subtraction, multiplication, division, the use of fractions and measurements as recorded in Egyptian Papyri were in connection with practical problems of the society (Zaslavsky, 20).

As is common place knowledge now, the great ancient Western mathematicians were at one time or the other under the tutelage of the scribes of the Egyptian Mystery Schools (Sarton IX). It need be noted here that as mathematics spread from Mesopotamia and Egypt independently to Italy, Ionia and Athens, its practical status was preserved. The transition from applied to pure or theoretical mathematics took place sometimes in the 15th century B.C. but it is still a matter of debate why and how this transition took place (Encyclopedia, 179). It has been suggested that this transition was necessitated by the discovery of the irrational numbers (Kneale & Kneale, 8).

The early Pythagoreans has postulated “that all things are numbers”. This might be taken to mean that any geometric whole number can be associated with some numbers (i.e some whole number of fraction, what is modern mathematical terminology would be rational number). This is so because in Greek usage then, *Arithmos*, which means “number” refers exclusively to whole numbers, and in some contexts to ordinary fractions. This assumption is common enough in practice, as when the length of a given line is said to be so many feet plus a fractional part.

This ancient Greek mathematical reasoning breaks down when applied to the lines that form the side and diagonal of a square.



If, for example, it is supposed that the ratio between the side and diagonal may be expressed as two whole numbers, it can be shown that both of these two whole numbers must be even. This is impossible since fraction may be expressed as a ratio of two whole numbers having no common factors. Geometrically, the fatal implication here is that there is no length that could serve as a unit of measure of both the side and the diagonal that is, the side and diagonal cannot each equal the same length multiplied by (different) whole numbers. The discovery then that the side and diagonal of a square are not susceptible of linear combination might have prompted the transition from applied to theoretical mathematics.

It has been said that Zeno of Elea, the staunch defender of monism and unity discovered the mathematical device of *reduction ad impossibile.* This method is also known as dialetics in the sense of drawing unwelcome consequences from initial hypothesis (Kneale & Kneale, 8-9). Zeno then posed what has come to be known as Zeno's paradoxes. In the paradox of the divided line, it is assumed that a line can be bisected again and again without limit; if the division ultimately results in a set of points of zero length, then adding this zero length infinitely would amount to nothing but zero still. On the other hand, if the division results in tiny line segments, then their sum will be infinite. The paradox lies then in the fact that the length of a given line must be zero and infinite.

Democritus and other atomists attempted to resolve Zeno's paradox of the divided line by postulating the indivisibility of the atom (*atoma* in Greek means 'indivisible'). But this attempt met with an insurmountable obstacle in 'incommensurability' as the Greeks called it (that is irrational numbers). The zenonian mathematical device of *reduction ad impossible* only advertently attests to the empirical character of early mathematics. The methodology proceeds by assuming an hypothesis and pointing out the unwelcome consequences of the conclusion that is drawn from the hypothesis. This would usually carry the character of *mudus tollens*:

$$\begin{array}{l} P \rightarrow q \\ \quad -q \\ \therefore \; -p \end{array}$$

So, whether an argument (as an organic unit of propositions) had an internal harmony or not, if an absurd consequence can be pointed out when its conclusion is correlated with existential facts, such an argument, using *reduction ad impossibile* as a mechanism of adjudication, would be adjudged as wrong. So, the litmus test for the correctness of an argument was its conclusion's compatibility with historic phenomena. But the discovery of the irrational number in the incommensurability of the side and diagonal of a square, which is a geometric fact, made early mathematicians look beyond experience. In pure mathematics, non-existent numeric entities are mathematical facts such as the imaginary units, the irrational number, √2, √-1, etc (Zeldovich and Yaglom 458-459).

The semantics was naturally going to change from correctness in applied mathematics to validity in pure mathematics. With the publication of Euclide's Elements, (which is a crystallization of hitherto mathematical attempts made of various problems), mathematics assumed an ivory tower status. If Bacon's passionate desire for utility and practical application of knowledge blinded him to the vast theoretical value of mathematics

(Sahakian, 129), not so for Galileo. Galileo was an Italian. Lack of progress in Italy in mathematics, as in other spheres, has been attributed to the opposition from the inquisition, which condemned Galileo in 1616 privately and publicly in 1633. Galileo published his *Mathematical Theory of Motion,* which was to be later proscribed, which represented a decisive advance over Bacon's method of inquiry, grew out of his use of empirical Pythogoreanism i.e., the application of mathematical to empirical facts about motion (Sahakian, 129). Galileo thus took the laws of nature and placed then in a universal mathematical form. In fact, Galileo's mathematicization of empiricism has been eulogized in very superlative terms of follows:

> Thus the world is indebted to Galileo for correcting the vagaries of empiricism by means of mathematical calculations, replacing the sterile Pythagorean number philosophy of the Humanistic period with an empirically valid mathematical theory (Sahakian, 120).

This satisfaction with mathematical truths persisted through the modern period of philosophy. The most avowed and consistent of all three British empiricists, David Hume, in spite of his incurable skeptical predilection, still showered encomiums on mathematics thus:

> When we run over libraries, persuaded of these principles, what havoc must we make? If we take in our hand any volume; of divinity or school metaphysics, for instance; let us ask, does it contain any abstract reasoning concerning quantity or number? No. Does it contain any experimental reasoning concerning matter of fact and existence? No. commit then to the flames: for it can contain nothing but sophistry and illusion (Hume, 300).

Descartes, the acclaimed father of modern philosophy, continued the application of mathematics into other fields. Unlike Galileo who mathematicized empiricism. Descartes would rather we spoke of mathematics of philosophy in the place of philosophy of mathematics. Christopher Clevious had, before Descartes, been impressed with mathematics and suggested that it be applied to other disciplines to give them the certitude of mathematics. Descartes was to achieve this purport. Are mathematical truths as certain and indubitable as have been represented?

The epistemic status of mathematical truths

It has been alleged that although mathematicians speak various "natural languages" around the world, their mathematical language is practically universal (Wilder, 285). This universal language must be the "analytic" language. An analytic statement is a statement that is true by virtue of the meanings of its terms (Halverson, 280). Pure mathematics would definitely be adorned with the honourable robes of analyticity. It would be remembered that Descartes had noted Thus:

> …reason already convinces me that I must withhold assent no less carefully from what is not plainly certain and indubitable than from what is obviously false: so the discovery of some reason for doubt as regards each opinion will justify the rejection of all (61).

Curiously though, a man who had doubted everything, including his own existence, while speaking very disdainfully of the physical sciences, extolled mathematics as follows:

> At this rate we might be justified in concluding that whereas physics, astronomy, medicine, and all other sciences depending on the consideration of composite objects, are doubtful; yet arithmetic, geometry, and so on, which treat only of the simplest

> and general subject-matter, and are indifferent whether it exists in nature or not, have an element of indubitable certainty (63).

Descartes thus expressed his regrets why upon such stable foundation no knowledge has been built. He is said to have been so impressed with mathematical certainty that he invented analytic geometry.

However, applied or pure, mathematical truths cannot be epistemologically branded as indubitably certain. It is important to note Descartes' confession "… I have accustomed myself to withdraw my mind from the sense; I have been careful to observe how little truth these is in our perceptions of corporeal objects…." (92). Even if Descartes might not have given a thought to it, he inadvertently cast aspersions on the certainty of applied mathematical truths. Applied mathematical truths are synthetic truths whose truth or falsity is verifiable through the sense. But synthetic verification cannot yield certain and absolute results (Stumpf, 314).

Very fundamentally again, even the pure mathematics, which is purported to be universally analytically verifiable, has been weakened with devastating sledge hammer of criticism. Against this backdrop, Descartes, the inventor of analytic geometry, could not have been right to think of geometric truths as truly stable and certain. Ayer writes:

> But Descartes, though he regarded mathematics as the paradigm of knowledge, was aware that it's a priori truths are not indubitable in the sense that he acquired. He allowed it to be possible that a malignant demon should deceive him even with respect to those matters of which he was the most certain (80).

Furthermore, in *Quine's Two Dogmas of Empiricism* he queries that the discrimination of statement into analytical and systematic ones is itself arbitrary.

> It is obvious that truth is general depends on both language and extra-linguistic fact. The statement "Brutus killed Caesar" would be false if the world had been different in certain ways but would also be false if the word "killed" happened rather to have the sense of "beggar". Thus one is tempted to suppose in general that the truth of a statement is somehow analyzable into a linguistic component and a factual component should be null; and these are the analytical statements. But, for all it's a priori reasonableness, a boundary between analytical and synthetic statements simply unempirical dogma of empiricists, a metaphysical article of faith (Quine 37).

With Quine's criticism above, it becomes clear that a purely analytic truth without any stricture of synthetic embellishment, whose respectability is attainable in 'every possible world', is a seeming insurmountable ideal. This is why Sahakian's comment the Galileo successfully attempted 'empirical pythagoreanism' (the application of the mathematical to empirical facts), becomes reasonable. But the fundamental fact is that at the point where analytic propositions are tainted with synthetic coloration, the former cease to be 'analytic' in that sense. And at that point again the truth or falsity of such a proposition would no longer be purely dependent on an interpretation of the meaning of the terms such as "a circle is a figure with a circumference".

The emergence, for instance, of other forms of geometry outside the celebrated Euclidean geometry like Bolyai's and Lobachevski's many parallels geometry, and Reimann's no parallels geometry (Salmon, 6) is a sure manifestation of the ununiversality of Eclidean geometry, in spite of its analytic status. In fact, Euclid's axiom of continuity has been subjected not to be susceptible of both analytic and synthetic verification (Waiswman, 47). Given the many upheavals that have ravaged the foundations of mathematics (Resnik, 14), it is hardly reasonable to swallow any doctrine of the certainty of mathematical truths hook, line and sinker. Descartes asserts that all certain knowledge is based upon two mental operations: *intuition*- which he called “the natural light or reason”- and *deduction* (Rader, 132). Through a process of logical transitivity, for instance, we have the following:

$P \rightarrow q$
$Q \quad r$
$R \rightarrow s$
$\rightarrow P$
:.S

If P is certain, then S will be certain. P’s certainty is by intuition which S’s certainty is by deduction. But the initial axiom(s), which Descartes claims are given by the natural light of reason constitute the wilderness of doubt of mathematical certainty. This is because the spectacle of the clashes of intuitive apprehension and the relativities of perspectives would not allow us lie in tranquility in the assumption that geometric axioms and postulates are as certain as they are represented.

Conclusion

Where then lies the indubitability of the Cartesian geometric and mathematical truths? How true are these truths? If truth is taken as a proposition derived through a rigorous deductive logical process, from initial axiom(s), then some geometric conclusions that are so rigorously deduced can be said to be true. On the other hand, however, if truth implies the truth of the totality of the organic propositions that form an argument, i.e. including the truth of propositions that form the premise(s), then Descartes’ postulation of the certainty of geometric truths must be accepted *cum granu salis.*

Works cited

Ayer, J. A. “I think, therefore I am”. *Descartes: A Collection of Critical Essays*. Ed. Willis Doney. New York: Doubleday, 1967.

Descartes, R. *Meditations on First Philosophy. Descartes Philosophical Writings*. Trans, T. P. Geach and E. Anscombe. Nairobi Thomas Nelson and Sons, 1954.

Encyclopedia Britannica, “History of Mathematics” Vol. 23.

Halverson, H. W. *A Concise Introduction to Philosophy*: America: McGraw-Hill, 1967.

Hume, D. *An Enquiry Concerning Human Understanding. The Enduring Questions*. Ed. Melvin Rader. New York: Holt, Rinehart and Winston, (279-200) 1976.

Kneale, W. & Kneale, M. *The Development of Logic*. Oxford: Oxford University Press, 1961.

Lunmkpin, B. Africa in the Mainstream of Mathematics History *Blacks in Science Ancient and Modern*. Ed. Sertima, I. V. *The Journal of African Civilizations*, April and November Issues, 5(1):2. 1983.

Quine, W. O. “Two Dogmas of Empiricism: *From a Logical Point of View: Logico-Philosophical Essays.* Cambridge. Harvard University Press, 1961.

Rader, M. (Ed). *The Enduring Questions.* New York: Holt, Rinehart and Winston, 1976.

Resnik, M. D. *Frege and the Philosophy of Mathematics.* Ithaca: Cornwell University Press, 1980.

Sahakian, W. S. *Outline-History of Philosophy*. New York: Barnes and Noble, 1968.

Salmon, W. C. *Space, Time and Motion: A Philosophical Introduction.* Minneapolis: University of Minnesota Press, 1980.

Stumpf, S. E. *Socrates to Sartre: A History of Philosophy.* 3rd Ed. America: McGraw Hill, 1982.

Waismann, F. *The Principle of Linguistic Philosophy*. London, Macmillan, 1965.

Zeldovich, Ya. B. & Yaglom, I. M. *Higher Math for Beginners.* Moscow: Mir, 1982.